Bayesian network modeling

Probabilistic vs. deterministic modeling approaches

Probabilistic

↑ Explanatory power (e.g., r^2)
↓ Explanation why
Based on *inductive reasoning*

Good when you:

- Want to explore for patterns
- Want to see if real-world patterns conform to theory
- Have incomplete datasets or high uncertainty

Pitfalls (among others):

- Putting too much faith into patterns found in the data that lack a reasonable theoretical foundation

Deterministic/mechanistic

↑ Explanation why
↓ Explanatory power (e.g., r^2)
Based on *deductive reasoning*

Good when you:

- Want to test/understand why something works the way it does
- Have strong understanding of how something works

Pitfalls (among others):

- Sloppy model construction

Steps in a typical modeling process

1. Define system boundaries
2. Define model elements/variables
3. Build conceptual model
4. Identify potential feedback loops, thresholds, equilibria
5. Collect & prepare data to parameterize model
6. Formalize mathematical relationships
7. Testing, validation, calibration, sensitivity analysis

Bayes' theorem

$$p(A|X) = \frac{p(X|A)*p(A)}{p(X|A)*p(A) + p(X|{\sim}A)*p({\sim}A)}$$

How do we update the probability of A when we get new evidence, X?

Bayesian Inference

Experiment Judy picks a jar at random, and then a cookie at random. The cookie is plain. What's the probability that Judy picked from jar #1?

Prior Probabilities $P(J_1) = P(J_2) = 0.5$

Event E = observation of plain cookie

Conditional Probabilities
$P(E|J_1) = 30/40 = 0.75$
$P(E|J_2) = 20/40 = 0.50$

Bayesian Inference

Experiment Judy picks a jar at random, and then a cookie at random. The cookie is plain. What's the probability that Judy picked from jar #1?

Bayes Theorem $P(J_1|E) = \dfrac{P(E|J_1)\,P(J_1)}{P(E|J_1)\,P(J_1) + P(E|J_2)\,P(J_2)}$

Posterior $P(J_1|E) = \dfrac{0.75 \times 0.5}{0.75 \times 0.5 + 0.5 \times 0.5} =$

Probability = **0.6**

Bayesian/probabilistic modeling

- Elements are assigned probabilities of occurrence (in the absence of data) – *conditional* and *prior* probabilities
- Data replace prior and conditional probabilities when available
- Provides results as a distribution of values without requiring stochiastic variables

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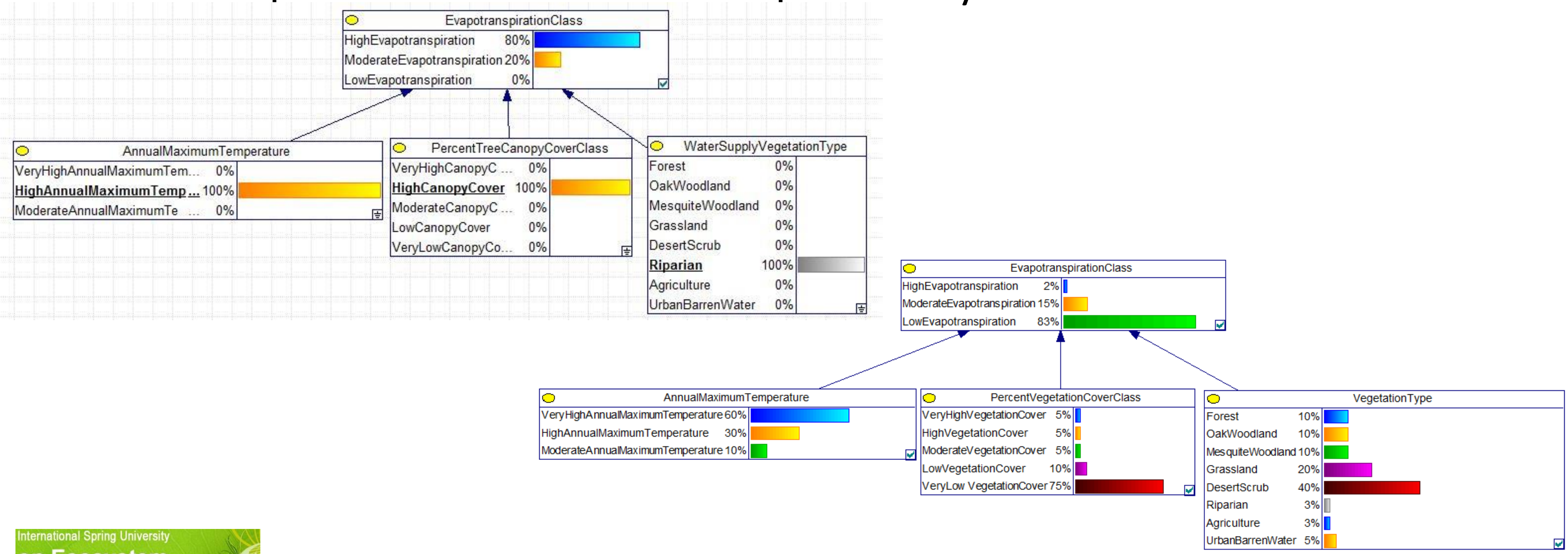
Uncertainty in deterministic models

- All else being equal (i.e., same input data & equations), you'll get the same results every time
- Change input parameters, use stochastic inputs & run repeatedly to generate a *distribution of results* (Monte Carlo simulation)

Uncertainty in probabilistic models

- Uncertainty estimates “built in” with prior probabilities & conditional probability tables



Guidelines for Bayesian modeling (Marcot et al. 2006)

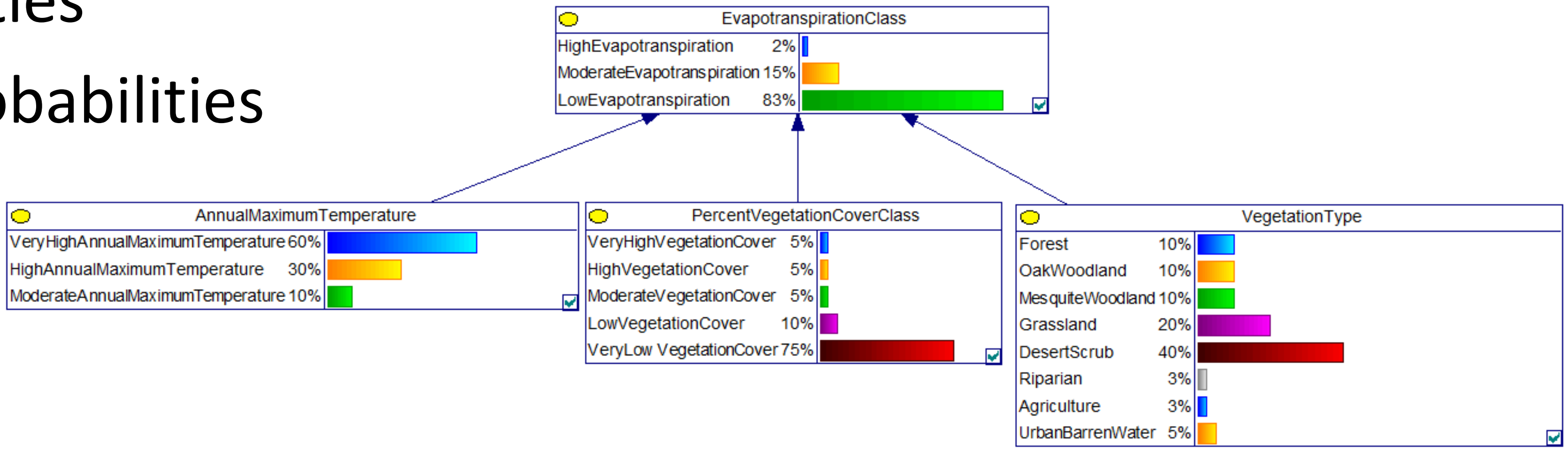
1. Develop causal model (i.e., influence diagram/directed acyclic graph)
2. Discretize each node
3. Assign prior probabilities
4. Assign conditional probabilities (“alpha-level model”)
5. Peer review (“beta-level model”)
6. Test with data and train the BN (“gamma-level model”)

General tips (Marcot et al. 2006)

- Keep # of input (parent) nodes & their # of discrete states tractable relative to each child node
- Role of intermediate variables
- Avoid unnecessarily "deep" models (problems with uncertainty propagation)
- Using training data
- CPTs: can use equations or "peg the corners;" potential role when thresholds are known

Building the mathematical model: Probabilistic models

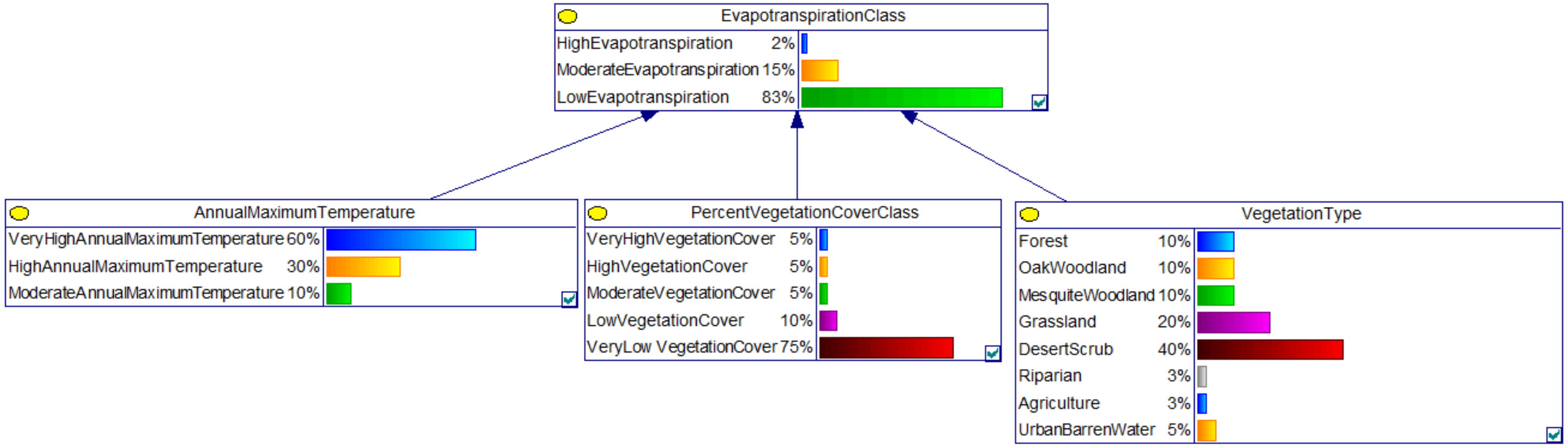
- Discretize variables
- Assign prior probabilities
- Assign conditional probabilities



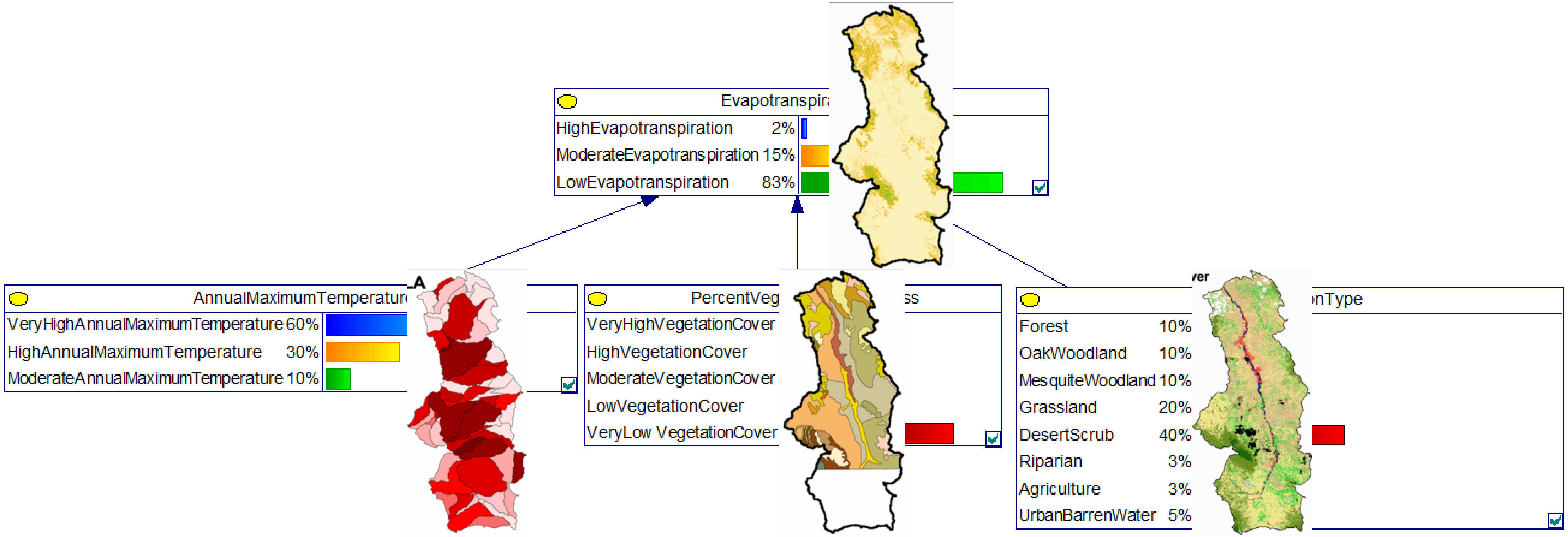
VegetationType	OakWoodland														
PercentVegetationCoverClass	VeryHighVegetationCover			HighVegetationCover			ModerateVegetationCover			LowVegetationCover			VeryLowVegetationCover		
AnnualMaximumTemperature	VeryHighA...	HighAnnual...	ModerateA...	VeryHighA...	HighAnnual...	ModerateA...	VeryHighA...	HighAnnual...	ModerateA...	VeryHighA...	HighAnnual...	ModerateA...	VeryHighA...	HighAnnual...	ModerateA...
HighEvapotranspiration	0.2	0.1	0.1	0.15	0.05	0.05	0.15	0.05	0.05	0.1	0.05	0.05	0	0	0
ModerateEvapotranspiration	0.7	0.7	0.6	0.7	0.75	0.7	0.6	0.6	0.5	0.45	0.4	0.3	0.4	0.3	0.2
LowEvapotranspiration	0.1	0.2	0.3	0.15	0.2	0.25	0.25	0.35	0.45	0.45	0.55	0.65	0.6	0.7	0.8

Bayesian network training

- Bayesian training: Process where the system quantifies the relative contribution of parent nodes to child node in a BN
- User-specified CPT becomes much less relevant



Bayesian network training



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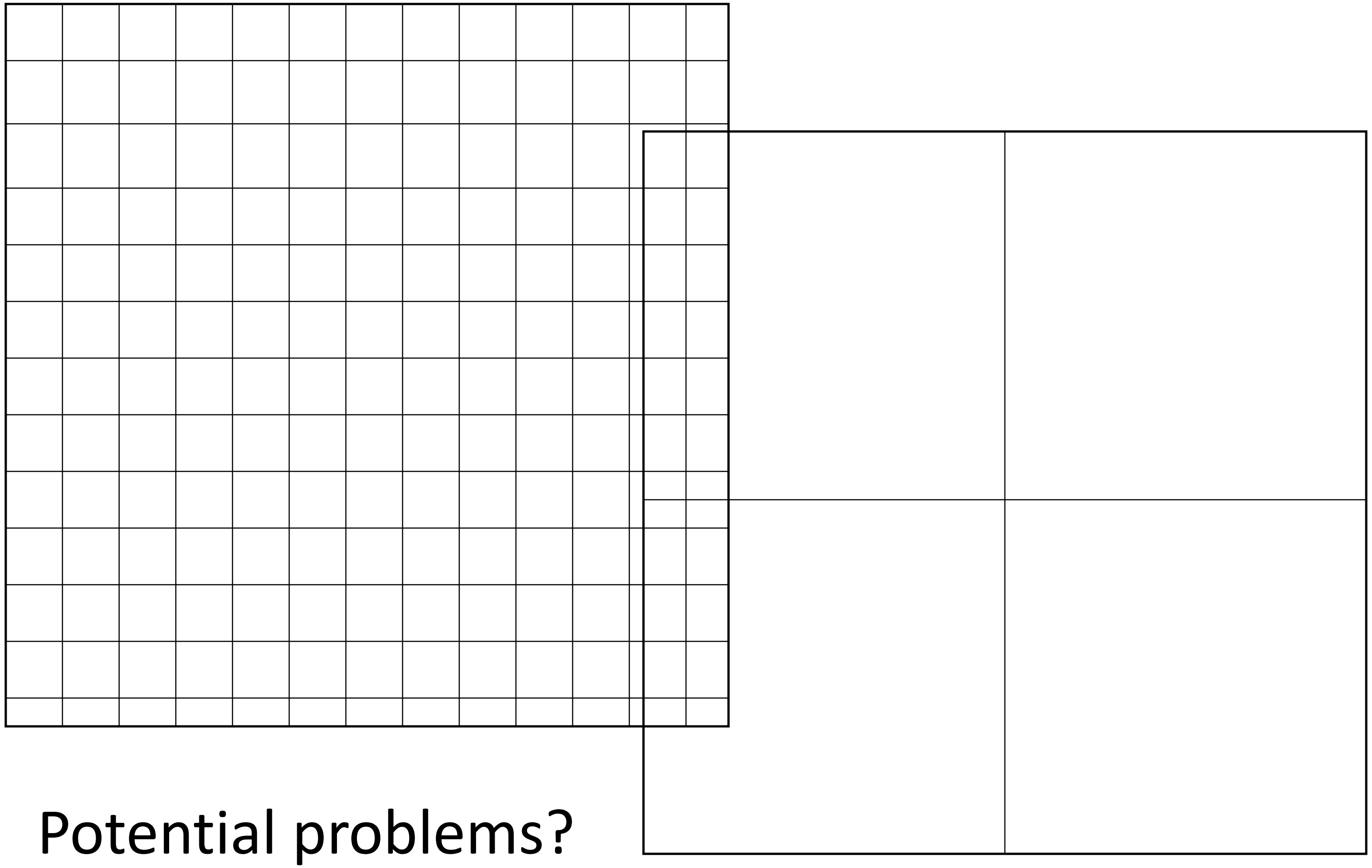
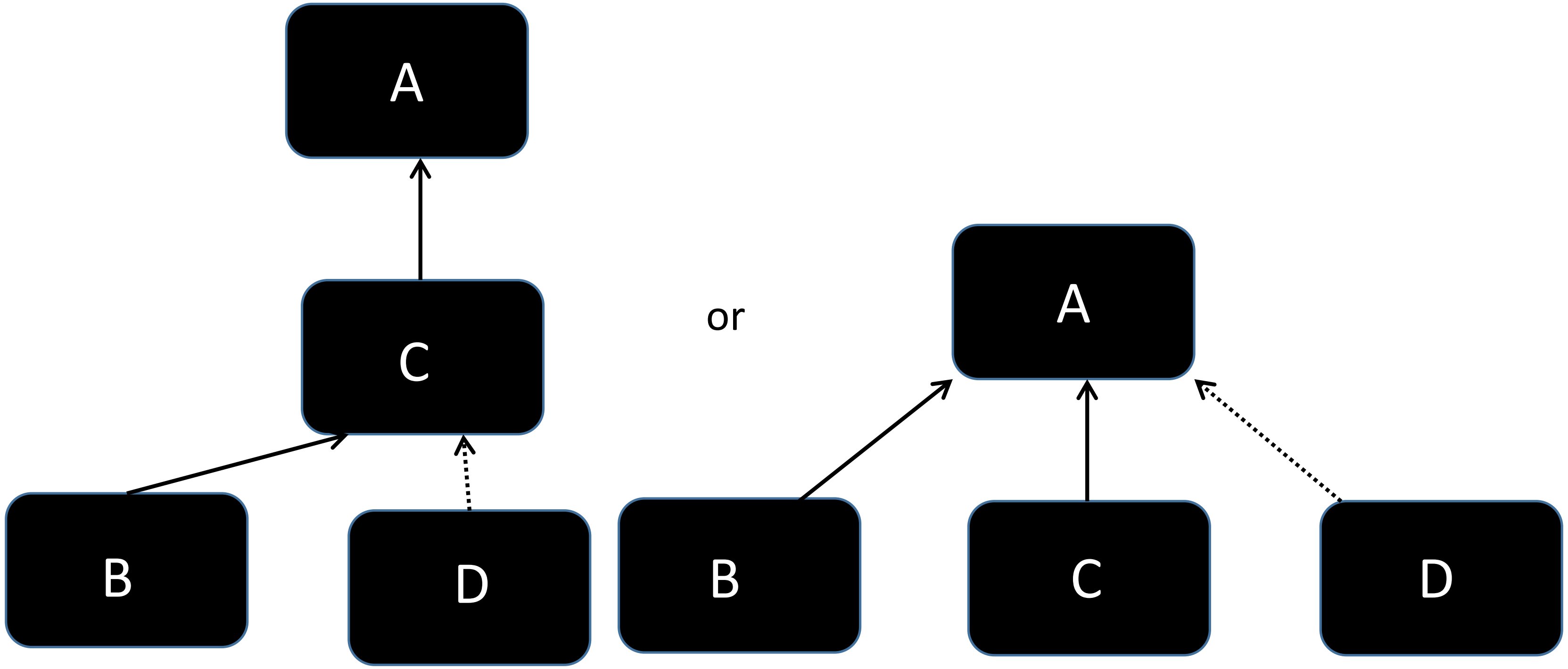
Spatial resolution & Bayesian network training



Potential problems?

What if the system could determine the optimal model structure?

- Structural learning



Parting words

- Ockham’s Razor/parsimony/KISS principle
- Start simple, continuously test the model, and add features/complexity slowly and carefully
- Keep your eye on the ball (original goals)
- Use best available data & assumptions
- Peer review is a good thing
- Document everything!

For more information

- http://yudkowsky.net/rational/bayes
- Pearl, J. 1988. Probabilistic reasoning in intelligent systems: Networks of plausible inference. Morgan-Kaufmann: San Francisco, CA.
- Marcot et al. 2006 & McCann et al. 2006 articles (distributed with course materials)

On Bayesian modeling

"Some would argue that incorporating beliefs about models other than those implied by empirical measurement is a subjective, or unscientific, approach. In response, it could be stated that, certainly, Bayesianism has the potential for this problem to arise, and so one must have a strict 'code of conduct' for prior distribution specification. For example, making use of the outcomes of previous studies to provide prior beliefs is a reasonable scientific standpoint. Indeed, it could be argued that it is unscientific to ignore these prior results! Another way of avoiding subjectivity is to use non-informative priors in cases where prior information is unavailable or unobserved. Of course, one could argue that even a non-informative prior gives us some form of information about the distribution of an unknown parameter: after all, a specific distribution is being supplied rather than the information that any distribution might apply. However, in many cases non-informative priors do make reasonable models for a state of no subjective knowledge. In several 'text-book' examples of Bayesian analysis, for example multiple linear regression analysis assuming normal error terms, the adoption of non-informative priors results in tests algebraically identical to classical inferential procedures. In most cases, analysts are reasonably satisfied with regarding such classical approaches as 'objective'."

- Brundson & Willis 2002

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Bayes' theorem: cancer screening example

Convert the plain English to mathematical notation:

1% of women over 40 that are routinely screened have breast cancer

$$p\,(c) = 0.01$$

80% of women with breast cancer test positive for cancer with a mammography

$$p\,(m+|c) = 0.8$$

9.6% of women without breast cancer also test positive for cancer with a mammography (false positive)

$$p\,(m+|{\sim}c) = 0.096$$

We want to know the likelihood of cancer, given a positive test

$$p\,(c|m+) = ?$$

Bayes' theorem: cancer screening example

$$p(c|m+) = \frac{p(m+|c)*p(c)}{p(m+|c)*p(c) + p(m+|\sim c)*p(\sim c)}$$

$P\ (c|m+) = (0.8*0.01)/[(0.8*0.01) + (0.096*0.99)] = 0.07764 = 7.8\%$

How do we update the probability of c when we get new evidence, m+?